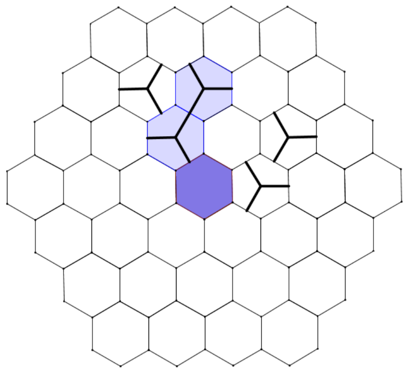
15 November 2013

Team olympiad

1. On the left you see a village map. The village consists of 37 hexagonal regions (each region is owned by a family). There is a source of water at the central region. There is a system of pipes in the village that is used to supply the people with water. Inside each non-central region the pipes are arranged in one of the two ways shown at the picture. The two ways differ by rotation by 60°. We say that water *comes* to a region if water can go from the source along pipes to the center of the region. Is it possible to arrange the pipes in such a way so that the water comes to every of the regions? (At the picture we give an example of how the pipes can be arranged in some regions. The water comes to the grey regions).



2. 10 people are standing in a row. Each person is either a knight or a liar. Knights always tell the truth, and liars always lie. Five people said: «There are exactly three knights to the right from me». The other five people said: «I have a left-hand neighbour and he is a knight». How many knights were there among the 10 people? (All possible answers should be found and a proof that there are no other answers should be given.)

3. Let $ABCD$ be a quadrilateral such that $\angle BCD = \angle ABC = 120°$, $BC + CD = AD$. Prove that $AB = CD$.

4. Baron Münchhausen has a set of 7 masses (some masses may be equal). Baron says that for any $k = 2, 3, 4, 5$ he can choose k masses from the set in such a way that their total mass is half the mass of all the masses in the set. Can this be true?

5. Find the minimum n such that the sum $1 + 11 + 111 + \cdots + \underbrace{111\ldots1}_{n}$ is divisible by 45.

6. There are one million boys and one million girls in a town. Some of them are friends and some are not. The Snow Queen can quarrel a boy and a girl, or make them friends, by saying 1 magic word. The Queen is happy only if each boy has an odd number of friends among girls and each girl has an odd number of friends among boys. What is the smallest number of magic words that is always sufficient for the Snow Queen to become happy?

7. Given $\frac{x-y}{1+y} + \frac{y-z}{1+z} + \frac{z-x}{1+x} = 1$, find $\frac{1+x}{1+y} + \frac{1+y}{1+z} + \frac{1+z}{1+x}$.

8. The 50 numbers 1, 2, …, 50 are written on the blackboard. In one step we can choose a number (which we denote by A), calculate how many numbers A are there on the blackboard, denote the result by N, and after that write N instead of A everywhere on the blackboard. In this way, we are allowed to make several steps. Can we get 50 numbers "4" on the blackboard?

9. In $\triangle ABC$, D is a point on line CB such that B is between C and D and $AB = BD$, and M is the midpoint of AC. Point P is the intersection point of line DM and the bisector of $\angle ABC$. Prove that $\angle BAP = \angle ACB$.

10. At the beginning, there were 8 people in the Tournament Committee. Each member has an opinion about each of the other members: he thinks that some of them are competent, and the others are not. These opinions never change. Every morning, the Committee votes: each member writes down the list of all members that are not competent (in his personal opinion). If a member is considered to be not competent by at least half of the Committee then this person is kicked out from the Committee (and the person is not a member of the Tournament Committee any more). Prove that on the 5th day nobody will be kicked out from the Committee.