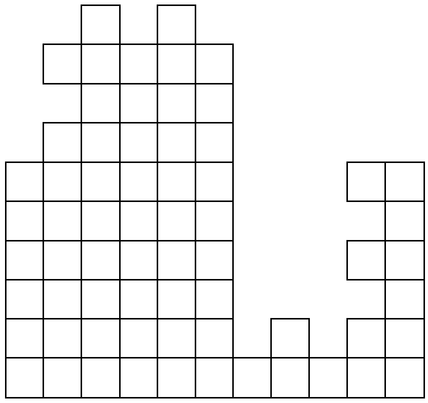
1st International Mathematical Tournament "DvazhdyDva" for Mixed-Age Teams

November 2, 2012

Team Olympiad

1. Cut the figure into two parts, that can be moved and rotated to make a rectangle. (Parts can not be overlaped and turned.)



2. Once upon a time there was an island in the sea inhabited only by knights (who always tell the truth) and liars (who always lie). One day three of the islanders made two statements each.

   A: «Not more than 3 people live on the island», «All the island inhabitants are liars».

   B: «Not more than 4 people live on the island», «Not all of the islanders are liars».

   C: «There are 5 people living on the island», «There are at least 3 liars on the island».

   How many people lived on that island and how many of them were liars?

3. The Magical Land has coins of 1, 2 and 3 Eyuns which are 1, 2 and 3 grams in weight respectively. Sasha has three coins of each type. One of these nine coins is false, its weight differs from a true coin's weight. How can Sasha find the false coin in three weightings on a beam balance without weights?
4. The chessman "Unicorn" moves one square to the right, down or diagonally to the left-up. It started on the square at the upper left corner, moved through the chess board, visited every square once. Can it finished on the square at the lower right corner?
5. Grigoryev's machine can change two cards with natural numbers *A*, *B* on them into one card with their sum or into *A* cards with number B on each of them. Initially Serezha has three cards with numbers 3, 4 and 5. He wants to get exactly one card with the number 2012. Will he cope with this task?
6. Kolya has a box sized 40x50x70 inches that is filled with balls. The radius of each ball does not exceed 5 inches. Prove that Kolya could shift the balls into the box sized 30x40x160 inches.
7. During a chess tournament each participant played with each other two games: one game with the white pieces and the second with the black ones. All the participants scored the same number of points (win is worth one point, tie is worth half a point, loss is worth zero points). Prove that there are two players that have won the same number of games with the white pieces.
8. The triangle ABC is inscribed in a circle. C_1 is a midpoint of an arc AB, A_1 is a midpoint of an arc BC, B_1 is a midpoint of an arc AC. Segment A_1C_1 intersects segments AB and BC at the points K and M respectively. Segment A_1B_1 intersects segments AC and BC at the points T and P respectively. Prove that angles KB_1M and TC_1P are equal.
9. Is it possible to put 1000 queens on a chessboard sized 2012x2012 squares so that they capture on all squares? (A queen is also considered to capture on its own square.)
10. There are *n* huge barrels in a row. There is 1 liter of milk in the first barrel, there are 3 liters of milk in the second etc. There are *2n-1* liters of milk in the last barrel. Winnie-the-Pooh can pour into any barrel as much milk as is already there. It is prohibited to pour milk out of a barrel. It is also prohibited to pour milk from one barrel into another if the first barrel has not enough milk for doubling milk in the second one. For which *n* is it possible to collect all milk in one barrel using these operations?